

# Promoting deaf pupils' achievement in mathematics <sup>1</sup>

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We describe in this paper an intervention project designed to raise the achievement of deaf pupils in mathematics. It is well established that deaf pupils lag behind hearing pupils in mathematics. The National Council of Teachers of the Deaf (1957) carried out a study with a large sample of deaf pupils in England and reported that deaf pupils were on average 2.5 years behind in mathematics achievement tests. About a decade later, Wollman (1965) reported similar results in a survey that included one third of the pupils from 13 schools for the deaf in the United Kingdom. Wood, Wood, and Howarth (1983) found that no improvement in this situation could be documented two decades later: in their study hearing impaired pupils were approximately 3.4 years behind in mathematics achievement when compared to their hearing counterparts. In the first section of this paper, we argue that it is possible to raise the mathematics achievement of deaf pupils. In the second section we describe the intervention programme we carried out. In the last section, we present the results of its assessment.

## *Is it possible to raise the mathematics achievement of deaf pupils?*

We (Nunes & Moreno, 1998) have argued that hearing loss cannot be treated as a cause of difficulties in mathematics but as a risk factor. Several findings in the literature suggest that hearing loss is not a direct cause of difficulties in mathematics. First, not all deaf pupils are weaker in maths than their hearing counterparts: approximately 15% of the profoundly deaf pupils perform at average or above average levels in standardised tests (Wood, Wood, and Howarth, 1983). If hearing loss were a direct cause of difficulties in mathematics, there should be no deaf pupils displaying achievements adequate for their age level.

Second, most studies have found either no correlation or only a very small correlation (Wood, Wood, and Howarth, 1983; Nunes & Moreno, 1998) between the level of hearing loss and mathematics attainment. This result suggests that hearing loss is not a direct cause of difficulties in mathematics.

We proposed the alternative hypothesis that hearing loss places children at risk for difficulties in learning mathematics. The difference between a risk-factor hypothesis and a causal hypothesis is that it is possible to prevent a risk factor from leading to negative outcomes if the necessary steps are put into place.

We identified two specific difficulties of deaf pupils that can explain, at least in part, why they are at risk for low achievement in mathematics.

First, deaf children have fewer opportunities for incidental learning as a consequence of their hearing loss. This is a hypothesis already proposed by Furth (1966) and Rapin (1986), who suggested that deaf youngsters' poor results in reasoning tasks and educational assessments can be explained by an 'information deprivation' (Rapin, 1986, p. 214). Deaf youngsters lack access to many sources of information (e.g., radio, conversations around the dinner table) and their incidental learning may suffer from this lack of opportunity. Consequently some concepts that hearing children learn incidentally in everyday life may have to be explicitly taught to deaf pupils in school. We (Nunes & Moreno, 1998) identified one mathematical concept – *additive*

*composition* – that is crucial to progress in mathematics, that is often mastered by children before they enter school or quite early on in their school lives, and that seems to create a significant obstacle for deaf children. Additive composition refers to the understanding that any number can be seen as the sum of other numbers. There is evidence that hearing children learn about additive composition informally, probably through their experiences with money, amongst other things. If a child is asked to pay for a sweet that costs 8 pence, for example, a child who understands additive composition has no difficulty in using one 5p and three 1p coins to pay the 8p. About 60% of 6-year-old and virtually all 7-year-old hearing children succeed in this task (Nunes & Bryant, 1996). In contrast, many deaf children, including some as old as 10 and 11 years (Nunes & Moreno, 1998), cannot combine coins of different values into a single amount.

A second difficulty of deaf children, often reported by parents (see Gregory, 1995), is related to communication about time. Research has shown (Moreno, 2000) that deaf children have significantly more difficulty than hearing children in making inferences that involve processing a sequence of events over time. This ability is often required in the mathematics classroom when we teach pupils about the inverse relation between addition and subtraction. Consider the problem ‘Mary had some sweets; her grandmother gave her 3 and now she has 8. How many sweets did she have to begin with?’ If pupils have difficulty in making inferences in connection with time sequences, it will be very difficult for them to work with problems that require them to think about inversion and time. In a previous study (Nunes & Moreno, 1997) we showed that deaf pupils were not behind hearing pupils when they solved problems that did not involve inversion but were considerably behind when the problems required such inverse inferences. Moreno (2000) found that deaf children’s performance in tasks that require making inferences about events over time is predictive of their mathematics achievement scores after an interval of seven months.

In order to promote the development of deaf pupils in mathematics, we developed a programme with two aims. The first was to give the deaf pupils opportunities to learn core mathematical concepts that many hearing pupils may learn informally outside school, and to promote connections between these informal concepts and mathematical representations used in school. The second aim was to promote deaf pupils’ access to information about word problems related to transformations over time by representing the problems through drawings and diagrams and reducing the need to retain information about sequences of events in memory.

### ***The programme***

The concepts that we Included in our intervention programme were:

- (1) additive composition and its application to number and measurement;
- (2) additive reasoning (that is, reasoning about the relations between addition and subtraction as inverse of each other);
- (3) multiplicative reasoning (that is, reasoning about the relations between multiplication and division);
- (4) ratio and fractions.

The programme was not envisaged as a replacement to the mathematics curriculum taught in the classroom. Its aim was to bring the deaf children’s informal mathematical understanding to a level where it could offer a solid basis for learning the curriculum that they are taught in school.

The programme was designed with the involvement of nine teachers of the deaf who attended monthly meetings with the researchers over five school terms. In the

Spring and Summer terms of 1998, the researchers prepared basic materials relevant to the teaching of each of the key concepts and discussed each set of materials with the teachers. The teachers tried out the materials and reported back on the results, positive features and the difficulties that the pupils had experienced. The materials were revised on the basis of the teachers' feedback.

The revised programme was organised in the booklets to be used in the classroom in the second phase of the project. The final number of teachers administering the programme in the classroom was six and the number of pupils involved was 24.

All teachers were asked to use the whole programme with all the pupils, even if they expected some of the concepts to be too easy or too difficult for their pupils. This proved to be a good measure as teachers often reported unsuspected competencies as well as difficulties amongst their pupils.

The teachers were encouraged to introduce new items with practical materials in order to ensure that the situations were understood by the children and to use discussions amongst the pupils as the items were solved. They provided their own explanations to the pupils. The programme booklets worked as motivation to work on everyday mathematical concepts. Some lessons were video-taped to provide us with feedback but no formal analysis of the tapes was carried out.

Observations and teachers' reports made it clear that there was much variation in the implementation. Some teachers promoted pupil interaction more than others. Some teachers allowed the children to identify mistakes themselves by comparing their work whereas other teachers provided feedback themselves. However, they all worked from the booklets and reported that the children enjoyed the work.

### ***Assessment of the programme***

Before the implementation of the programme, the pupils were assessed in the NFER-Nelson age appropriate mathematics tests (beginning Autumn/98). The programme was administered by the teachers at their own pace, making use of time normally scheduled for mathematics lessons over two terms (end of Autumn/98, Spring and beginning Summer/99). During this period, the teachers continued to attend monthly meetings with the researchers to discuss the proposed materials before application and report on the implementation of the materials used during the previous month.

At the end of the programme, the pupils were assessed in the same NFER age appropriate mathematics test that they had answered in the Autumn term as well as the new age-appropriate test if the children had had a birthday after the pre-test.

### **The programme**

#### Overall structure

Each concept was explored by means of a series of tasks. In each series, the items were ordered according to their expected level of difficulty. The most difficult items in one series were often more difficult than the easier items in the following series. This gave the children a sense of progress in one series and also the feeling that some easier tasks would be coming later on. The work with each concept is briefly described in the following sections.

#### Additive composition, number and measurement

The aims of this section of the programme were:

- to strengthen the pupils' understanding of additive composition;

- to strengthen their understanding of how numbers are used to measure, thereby expanding of the use of additive composition;
- to introduce the number line as a working tool for representing and solving problems.

*Items on additive composition*

Two examples are included in [Figure 1](#).

The aim of the items was to use pupils' informal knowledge of money and strengthen their understanding of additive composition. We discussed with the teachers the difficulties of each type of item and different ways of promoting children's understanding of the crucial concepts. For example, pupils who have difficulty with additive composition benefit from representing the value of non-unitary coins (5p or 10p) with their fingers before counting the total sum of money. They also benefit from working with values where the number words facilitate the task – for example, combining a 20p coin with 1p coins – before working with values where the number words do not help – for example, combining 10p and 1p coins. When pupils succeed in these simpler tasks, they can go on to more difficult items and be asked to think about how they solved the previous items.

*Items on measurement*

We saw two advantages of including items on measurement in this series of tasks related to number concepts. First, it is important that pupils think about the quantities represented by numbers in order to strengthen their understanding of number. Our previous work indicated that pupils' understanding of measurement is often incomplete. When measuring, many are not sure where to start measuring from –the edge of the ruler, zero or one. This suggests that they do not fully understand what the reading obtained from a ruler indicates. Second, rulers can be used as a number line. Thus they are useful in strengthening pupils' familiarity with mathematical conventions. Some of the items involved measurement with a broken ruler. Measuring with a broken ruler reinforces pupils' understanding that it is possible to work with number lines that start at any point. Some items from this section are presented in [Figure 2](#).

Our discussions with the teachers revealed that even some of their older pupils were not aware of the precise meaning of the numbers obtained from reading a ruler. The examples in [Figure 2](#), right, show how a pupil counts the 'longer lines' on the ruler as indication of the number of centimetres and the shorter lines as indication of the number of half centimetres. The pupil counted the lines, not the units of length.

*Items for introducing work with the number line*

Work with the number line was included in the programme for two reasons. First, it is a form of conventional mathematical representation. Second, the number line offers a visual representation of number sequences. It is known from previous work that deaf adults process information presented visually more efficiently than information presented orally. Thus we expect the number line to be a useful tool for deaf children when they calculate or discuss numerical information in the classroom. Two examples of tasks used to introduce the number line are presented in [Figure 3](#).

The teachers reported that the children did not find it difficult to use the number line to represent an answer. This facilitated the transition to using it as an instrument in problem-solving.

### *Additive reasoning*

The aims of the section on additive reasoning were:

- to promote the co-ordination of addition and subtraction as inverse of each other;
- to work with drawings and diagrams, representing time through spatial relations;
- to use the number line for calculation and for the demonstration of different solutions to the same problems.

Problems involving addition and subtraction with different number meanings and different levels of complexity (invisible addends, start unknown, comparison) were included to provide the pupils with the opportunity to explore additive reasoning broadly. Teachers were encouraged to use the pupils' records for the discussion of different ways of solving the same problems. Some examples of items are presented in [Figures 4 and 5](#).

The classroom observations showed that some teachers used concrete materials to help the younger children reason about problems that they found more difficult. Once the pupils had solved some problems using objects, they were able to work with the booklet without difficulty. The connection between the way in which they had counted the objects and counting on the number line was easily made.

Problems with start unknown (Mary had some sweets; her friend gave her 2; now she has 8; how many did her friend give her?) and with transformation unknown (A boy had 5 cakes; he ate some; now he has 3; how many did he eat?) are quite difficult for deaf pupils as they involve making time-related inferences. One example is included in [Figure 5](#).

Comparison problems are the most difficult of the additive problems with natural numbers for hearing and deaf pupils alike. Our programme introduced comparison problems by initially connecting the comparison to additive transformations. Our previous research (Nunes & Bryant, 1996) has shown that this is an effective way to make the solution of comparison problems accessible to 6-year-olds. One example is shown in [Figure 5](#). Several were included in the programme. Teachers reported that the number line was a particularly useful instrument when discussing the logic of comparisons.

### *Multiplicative reasoning*

The aims of the section on multiplicative reasoning were:

- to work from pupils' intuitive understanding of correspondence as the basis of multiplicative reasoning;
- to work with drawings and diagrams that represent two variables in the representation of multiplicative concepts;
- to introduce tables and graphs as mathematical representations for multiplicative relations.

Some examples are presented in [Figures 6 and 7](#).

Much research has shown that pupils' misconceptions in the domain of multiplicative reasoning are rooted in the idea that multiplication is simply repeated addition (for a review, see Greer, 1992). We (Park & Nunes, 2000) have carried out an intervention study with hearing children aged 6 and 7 years, comparing instruction on multiplication as repeated addition and as an operation linked to the schema of correspondence. Children instructed through the correspondence schema performed significantly better in the post-test than those instructed through repeated addition. Some examples of multiplicative reasoning problems are included in [Figure 6](#). The two

items on top exemplify the connection between the correspondence schema and the concept of multiplication. Tables are used as a way of representing the connection between the variables. The bottom items illustrate how the connection between multiplication and division was introduced. The problem on the left is: The teacher had birthday; all the children in the class brought him two flowers; the teacher received 24 flowers; how many children are in his class? The problem describes a typical multiplication situation with a missing factor (number of children) and provides information about the product (total number of flowers). The pupils typically used a correspondence strategy to solve this problem: they circled the flowers in groups of two; each group corresponds to one child in the class. The problem on the right presents an incomplete table where the ratio of flowers per vase has to be identified by division.

The items used to introduce graphs and their connection to multiplication were designed to help the pupils make a transition from more concrete drawings to more abstract diagrams. The graphs were always presented as representations of problems. In the initial problems, the pupils represented their answers in the graphs. Later, they were asked to obtain information from the graph. Four problems are presented in [Figure 7](#).

An analysis of the pupils' productions showed that even the younger pupils were able to have some success in reasoning about and representing multiplicative problems with tables and graphs.

#### *Ratio and fraction materials*

The aims of the section on fractions were:

- to work with sharing and division as an intuitive starting point for reasoning about fractions;
- to promote a connection between pupils' understanding of fraction and division ( $1/3$  seen as one divided into three), and between fraction and ratio (one out of three means the same as a one-to-two ratio).

Figure 8 presents two examples, one of an item that connects sharing with fractions and a second that connects ratio and fraction.

The concept of fraction as traditionally taught (areas of a figure) was not stressed in the programme. Although it is often used in the classroom, research suggests that children's intuitions about sharing and division can provide a solid start for understanding fractions from about age 8 (Streefland, 1997). Children at this age already realise that one pie shared amongst two people will give larger pieces than one pie shared amongst three people. This helps them understand that  $1/2$  is a larger number than  $1/3$ . A common misconception about fractions documented in the literature is to think that  $1/3$  is larger because 3 is more than 2.

The teachers found the fraction items difficult to work with as they are not used to thinking about the connection between ratio and fraction. They also reported that the pupils found this section of the programme more difficulty than the previous ones but pointed out that the easier items were accessible even to the youngest pupils.

#### **Results**

The effect of the intervention was assessed in two ways. Firstly, the project pupils were compared to a baseline group composed of deaf children drawn from the same schools and tested on the NFER age appropriate tests in the previous year. This comparison was carried out both in the pre- and post-test. There is no reason to expect a

significant difference between the project pupils and the baseline in the pre-test. If the intervention was successful, the project pupils should perform significantly better than the baseline pupils in the post-test. Figure 9 shows two graphs displaying the comparison between the baseline and the project pupils: the top graph shows the comparison for the pre-test results and the bottom graph shows the comparison for the post-test results.

Statistical analyses showed that the baseline and the project pupils did not differ significantly at pre-test. At post-test, however, the project pupils performed significantly better than the baseline pupils. Thus we conclude that the project pupils improved significantly in their mathematics achievement during the time they were engaged in the programme.

Secondly, the project pupils' observed progress on the standardised mathematics assessment was compared to their expected progress. If the project was successful, their performance in the post-test should be better than the performance expected from them on the basis of their pre-test performance. The NFER norms for the age appropriate mathematics tests include a prediction of performance on tests administered at a subsequent age taking into account performance on a first testing occasion. These guidelines are produced for hearing, rather than deaf children. Pupils whose observed performance is at a lower level than the predicted score are said to have made less progress than expected. Those pupils whose observed and predicted performances coincide are said to have made average progress. Finally, pupils whose observed score is superior to the predicted score are said to have made more progress than expected. By using the NFER guidelines, we obtained predicted scores for each pupil and compared these predicted scores to the observed scores. Results showed that 31.8% of the pupils had observed scores lower than the predicted ones; 68.2% had higher observed than predicted scores. A Wilcoxon test for dependent samples showed that the difference between the predicted and observed scores was significant. The pupils' performance at post-test was thus significantly better than it would have been if their progress had been equivalent to the average amount of progress expected for hearing pupils during one school year.

### ***Conclusion and discussion***

The quantitative analyses suggest that the intervention was effective in increasing the deaf pupils' access to the mathematics curriculum. The effects we documented cannot be explained solely in terms of the teaching which the children would have normally received in the classroom because the baseline pupils had been exposed to the same curriculum in the same schools in the previous year. No major policy changes in the teaching of mathematics were introduced during this year in the schools.<sup>2</sup> The teachers did not increase the amount of time dedicated to teaching mathematics because the project was implemented during the time normally scheduled for mathematics lessons. No extra classroom helpers were made available during the project. Thus there is no other reason beyond their participation in the project that would lead us to expect that the pupils would perform significantly better than the baseline group nor that they would make more progress than predicted for hearing children during the year.

It is not possible to tease out the effects of the two types of experience provided by the programme, namely the school teaching of concepts that hearing children seem to acquire informally and the use of drawings and diagrams for supporting communication. Although the question of specific effects should be addressed in future research, it would have been outside the aims of this intervention, which were to

maximise the learning opportunities for the deaf pupils. We believe that many factors change when a programme such as this is introduced in the classroom and that experimental interventions outside the classroom are needed to separate out the effects of different aspects of an educational programme.

It is likely that, in this case, the effects should be attributed both to cognitive and motivational factors. The cognitive effects are likely to be based both on the specific design for teaching the core concepts included in the programme and on the use of drawings and diagrams in teaching. The instruction programme was carefully designed to make most use of children's mathematical intuitions and use them to confront conceptual difficulties identified in previous research. The programme also provided the pupils with tools for representing the core concepts in a mathematically adequate form. The use of drawings and diagrams was chosen for its potential in addressing the communication needs of deaf pupils. The pupils seem to have found drawings and diagrams useful as a means of representing their ideas and working towards solutions. The teachers reported that their pupils, after starting to work with the project booklets, had begun to use drawings and diagrams at other moments in their mathematics lessons.

We suggest that there were also motivational effects operating beyond these cognitive factors. Our own observations and the teachers' reports indicate that the pupils enjoyed working with the booklets. In one class they actually celebrated the moments when the teacher asked them to bring out the booklets: they clapped and showed great enthusiasm verbally. In another class the pupils did not want to interrupt their work at lunch time because they were too engaged in a discussion about measuring with a broken ruler – an event very unusual in the teacher's experience with this group of pupils.

Further research analysing specific aspects of this intervention is necessary to identify the most crucial cognitive elements in the project. However, the motivational effects obtained when pupils find out that they can succeed in mathematics are likely to be confounded with the cognitive effects in any successful project.

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<sup>2</sup> After the project was concluded, the English government introduced The Numeracy Hour in the schools. There is no overlap between the implementation of this programme and The Numeracy Hour.